# The Riemann Hypothesis 2

(1) $\quad 1-\frac{1}{2^s}-\frac{1}{3^s}-\frac{1}{5^s}+\frac{1}{6^s}-\frac{1}{7^s}+\cdots\ =$

For example if $s=2, (1)=\frac{6}{\pi^2}$

If the terms in (1) are ordered correctly then this is equivalent to through the primes:

(2) $\quad \frac{1}{2}\,\frac{2}{3}\,\frac{3}{5}\,\frac{6}{7}\ldots = 0$ when s = 1

(1) is conditionally convergent so if the order of the terms in (1) is incorrect then $(1) \neq (2)$. For example if a positive term was always alternated by a smaller negative term then an infinite number of terms arranged like this could not equal zero. In (1) each term is a fraction smaller than the previous term in the sequence.

To prove that N terms in (1) = 0 as N $\rightarrow\ \infty$, for s = 1 and for s = 0.

For s = 0 this becomes equivalent to the Mobius Function, as a term with an even number of factors becomes +1, with an odd number of factors becomes -1 and terms with squares are not in the series, and thus equal zero. For example $-\frac{1}{6^s}=-\frac{1}{1}$ where s = 0.

In (1), ½ the terms have 2 as a factor and each term with 2 as a factor is ½ the size of one that does not have 2 as a factor. For example $+\frac{1}{6}$ is half the size of $-\frac{1}{3}$. To see this one can imagine the terms of (1), all positive and marked off on a ruler. Comparing each odd term to the same term times 2, i.e. $\frac{1}{3}$ and $\frac{1}{6}$ the even term is half the distance from the start of the ruler as the odd term.

The terms of (1) have no squares, the ratio of terms in (1) is X:Y where X is those with an odd number of factors and Y with an even number of factors. To prove that in (1) X:Y = 1:1. In (3)

(3)

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \dots$$

The terms containing squares are included, such as $\frac{1}{4}, \frac{1}{8}$, … Consider first the terms containing no squares, and the terms containing squares are temporarily removed, shown in (4)

(4)

$$1 + \frac{1}{2} + \frac{1}{3} + 0 + \frac{1}{5} + \frac{1}{6} + + \frac{1}{7} + 0 \dots$$

(4) is the same as (1) with all terms positive, the zeroes corresponding to terms containing squares. Then re add all the non square terms times 4 in their original position as they occur in (3). For example

(5) $\frac{1}{4}, \quad \frac{1}{8}, \quad \frac{1}{12}, \quad \frac{1}{20}, \dots$

The ratio X:Y would now be the same in (4) as in (1) because each term, with an odd or even number of factors was multiplied by 4 and re added to (3) in the same position in the sequence it was temporarily removed from. Also if one multiplied all the terms in (1) by 4, the ratio of X:Y would remain the same. Mixing together two sequences with the same X:Y ratio so each retains this ratio X:Y gives the same ratio in (4) of X:Y. In terms of probability as in my first paper if one selected non square terms from (3) one would select those with an odd or even number of factors in the ratio X:Y, and those with 4 as a factor also in the ratio of X:Y, so selecting from both would still give a ratio of X:Y.

All the terms containing squares are re added to (4) in their original positions and in the process the ratio X:Y remains the same, (4) is now restored as (3). If the odd terms in (3) had a ratio of terms that were odd to those that were even of A:B, then the even terms in (3) would have a ratio of B:A because the even numbers have the same factors as the odd terms but with the factor of 2 added. $\frac{1}{2}$ the terms are even in the integers because they alternate odd and even then whatever A and B are the terms in (3) with an odd or even number of factors will be equally dense. Therefore X:Y in (1) is 1:1.

For $s = 0$ then, the even and odd terms are equally dense, like the odd and even numbers are equally dense in the integers. For example in the integers the even numbers are double the odd numbers, making them equally dense. The inverse of the integers would mean the even numbers are half the size of the odd number fractions and so again they are equally dense, alternating from odd to even with the fractions in order of decreasing size. So for $s = 0$, $(1) \to 0$ as $N \to \infty$ because as in my first paper on the Riemann Hypothesis if the positive and negative terms are equally dense, i.e. equally likely to be selected randomly, then they are like coin tosses that are equally likely to occur, so $(1) = 0$ and grows no larger than

$$(6) \qquad O\,N^{\frac{1}{2}+\varepsilon} \text{ as } N \to \infty$$

For $s = 1$ in (1), if one looks at the odd terms in the first N terms and twice those odd terms in the first N terms, and ignore all other terms one can see that the odd terms sum to a number and the even terms sum to ½ that number with the opposite sign. This is because each even term is ½ the size of its corresponding odd term and the opposite sign. So the even terms have the same rate of growth of their sum as the odd terms because though they are half the size they are twice as dense in (1).

In practise in the first $N$ terms of $(1)$ there would be other terms, and so the first $N$ terms of $(1)$ would vary in its sum as $N$ grew. The growth of the positive terms however is balanced by the growth of the negative terms because with any growth in the sum of the odd terms , the sum of the negative terms would have the same growth if that rate of growth continued in the same way.

If the odd terms sum to $X$ for the first $N$ terms then generally the rate of growth would be expected to be around $-X$ for the even terms if they continued their same rate of growth to $N$. So if the odd terms $\rightarrow X$ as $N \rightarrow \infty$ then the even terms $\rightarrow -X$ as $N \rightarrow \infty$ so $(1) = 0$. If the odd terms $\rightarrow \infty$ as $N \rightarrow \infty$ then the even terms also go to $-\infty$ as $N \rightarrow \infty$, the even terms half the size and twice as dense so $(1) = 0$. If the odd terms go to $0$ as $N \rightarrow \infty$ then so will the even terms and again $(1) = 0$. In all cases then $(1) = 0$.